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GATE question paper - Mathematics 2009 (MA)

Notations and Symbols used

$X \setminus Y$	:	$\{x \in X : x \notin Y\}$
Z	:	The set of all integers
Q	:	The set of all numbers
R	:	The set of all real numbers
C	:	The set of complex numbers
R^n	:	$\{ (x_l, \ldots\ldots, x_n) : x_i \in R \text{ for } 1 \le i \le n\}$
ℓ^1	:	The vector space of all sequences $\{x_n\}$ in C such that $\sum \lvert x_n \rvert < \infty$
c_∞	:	The vector space of all sequences $\{x_n\}$ in C such that $x_n \neq 0$ for at most finitely many values of n
$\lVert \cdot \rVert_p$	:	The p-norm for $1 \le p < \infty$
A^T	:	The transpose of the matrix A
U (a, b)	:	Uniform distribution on the interval (a, b)
$f[x_0 \ldots\ldots x_k]$	:	k th divided difference of f at $x_o \ldots\ldots x_k$
$\binom{n}{r}$	:	$\dfrac{n!}{r!(\overline{n}-r)!}$
E(X)	:	Expectation of the random variable X

Q. 1 -Q. 20 carry one mark each.

1. The dimension of the vector space $V = \{ A = (a_{ij})_{n\times n} : a_{ij} \in C, a_{ij} = -a_{ij} \}$ over the field R is

(A) n^2 (B) $n^2 - 1$ (C) $n^2 - n$ (D) $\dfrac{n^2}{2}$

2. The minimal polynomial associated with the matrix $\begin{bmatrix} 0 & 0 & 3 \\ 1 & 0 & 2 \\ 0 & 1 & 1 \end{bmatrix}$ is

(A) $x^3 - x^2 - 2x - 3$ (B) $x^3 - x^2 + 2x - 3$
(C) $x^3 - x^2 - 3x - 3$ (D) $x^3 - x^2 + 3x - 3$

3. For the function $f(Z) = \sin\left(\dfrac{1}{\cos(1/z)}\right)$, the point z = 0 is

(A) a removable singularity (B) a pole
(C) an essential singularity (D) a non-isolated singularity

4. Let $f(z) = \sum_{n=0}^{15} z^n$ for $z \in C$. If $C : |z - i| = 2$ then $\oint_C \dfrac{f(z)dz}{(z-i)^{15}} =$

(A) $2\pi i(1 + 15i)$ (B) $2\pi i(1 - 15i)$ (C) $4\pi i(1 + 15i)$ (D) $2\pi i$

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5. For what values of α and β, the quadrature formula $\int_{-1}^{1} f(x)dx \approx \alpha f(-1) + f(\beta)$ is exact for all polynomials of degree ≤ 1?

(A) $\alpha = 1, \beta = 1$ (B) $\alpha = -1, \beta = 1$ (C) $\alpha = 1, \beta = -1$ (D) $\alpha = -1, \beta = -1$

6. Let $f : [0, 4] \to R$ be a three times continuously differentiable function. Then the value of $f[1, 2, 3, 4]$ is

(A) $\frac{f''(\xi)}{3}$ for some $\xi \in (0,4)$ (B) $\frac{f''(\xi)}{6}$ for some $\xi \in (0,4)$

(C) $\frac{f''(\xi)}{3}$ for some $\xi \in (0,4)$ (D) $\frac{f''(\xi)}{6}$ for some $\xi \in (0,4)$

7. Which one of the following is TRUE?

(A) Every linear programming problem has a feasible solution.
(B) If a linear programming problem has an optimal solution then it is unique.
(C) The union of two convex sets is necessarily convex.
(D) Extreme points of the disk $x^2 + y^2 \leq 1$ are the points on the circle $x^2 + y^2 = 1$.

8. The dual of the linear programming problem :

Minimize $c^T x$ subject to $Ax \geq b$ and $x \geq 0$ is

(A) Maximize $b^T w$ subject to $A^T w \geq c$ and $w \geq 0$

(B) Maximize $b^T w$ subject to $A^T w \leq c$ and $w \geq 0$

(C) Maximize $b^T w$ subject to $A^T w \geq c$ and w is unrestricted

(D) Maximize $b^T w$ subject to $A^T w \geq c$ and w is unrestricted

9. The resolvent kernel for the integral equation $u(x) = F(x) + \int_{\log 2}^{x} e^{(t-x)} u(t)dt$ is

(A) $\cos(x - 1)$ (B) 1 (C) e^{t-x} (D) $e^{2(t-x)}$

10. Consider the metrics $d_2(f, g) = \left(\int_a^b |f(t) - g(t)|^2 dt \right)^{1/2}$ and $d_\infty(f, g) = \sup |f(t) - g(t)|$ on the space $X = C[a, b]$ of all real valued continuous functions on $[a, b]$. Then which of the following is TRUE?

(A) Both (X, d_2) and (X, d_∞) are complete.
(B) (X, d_2) is complete but (X, d_∞) is NOT complete.
(C) (X, d_∞) is complete but (X, d_2) is NOT complete.
(D) Both (X, d_2) and (X, d_∞) are NOT complete.

11. A function $f: R \to R$ need NOT be Lebesgue measurable if

(A) f is monotone
(B) $\{x \in R: f(x) \geq \alpha\}$ is measurable for each $\alpha \in Q$
(C) $\{x \in R: f(x) \geq= \alpha\}$ is measurable for each $\alpha \in Q$
(D) For each open set G in R, $f^{-1}(G)$ is measurable

12. $\{e_n\}_{n=1}^{\infty}$ be an orthonormal sequence in a Hilbert space H and let $x\ (\neq 0) \in H$. Then

(A) $\lim_{n\to\infty} \langle x, e_n \rangle$ does not exist (B) $\lim_{n\to\infty} \langle x, e_n \rangle =$

(C) $\lim_{n\to\infty} \langle x, e_n \rangle = 1$ (D) $\lim_{n\to\infty} \langle x, e_n \rangle = 0$

13. The subspace $Q \times [0, 1]$ of R^2 (with the usual topology) is

(A) dense in R^2 (B) connected (C) separable (D) compact

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14. $Z_2[x]/\langle x^3 + x^2 + 1\rangle$ is
(A) a field having 8 elements (B) a field having 9 elements
(C) an infinite field (D) NOT a field

15. The number of elements of a principal ideal domain can be
(A) 15 (B) 25 (C) 35 (D) 36

16. Let F, G and H be pairwise independent events such that $P(F) = P(G) = P(H) = \frac{1}{3}$ and $(F \cap G \cap H) = \frac{1}{4}$. Then the probability that at least one event among F, G and H occurs is
(A) $\frac{11}{12}$ (B) $\frac{7}{12}$ (C) $\frac{5}{12}$ (D) $\frac{3}{4}$

17. Let X be a random variable such that $E(X^2) = E(X) = 1$. Then $E(X^{100}) =$
(A) 0 (B) 1 (C) 2^{100} (D) $2^{100} + 1$

18. For which of the following distributions, the weak law of large numbers does NOT hold?
(A) Normal (B) Gamma (C) Beta (D) Cauchy

19. If $D \equiv \frac{d}{dx}$ then the value of $\frac{1}{(xD+1)}\left(x^{-1}\right)$ is
(A) $\log x$ (B) $\frac{\log x}{x}$ (C) $\frac{\log x}{x^2}$ (D) $\frac{\log x}{x^3}$

20. The equation $(\alpha xy^3 + y\cos x)\,dx + (x^2y^2 + \beta \sin x)\,dy = 0$ is exact for
(A) $\alpha = \frac{3}{2}, \beta = 1$ (B) $\alpha = 1, \beta = \frac{3}{2}$ (C) $\alpha = \frac{2}{3}, \beta = 1$ (D) $\alpha = 1, \beta = \frac{2}{3}$

Q. 21 to Q. 60 carry two marks each.

21. If $A = \begin{bmatrix} 1 & 0 & 0 \\ i & \frac{-1+i\sqrt{3}}{2} & 0 \\ 0 & 1+2i & \frac{-1-i\sqrt{3}}{2} \end{bmatrix}$
then the trace of A^{102} is
(A) 0 (B) 1 (C) 2 (D) 3

22. Which of the following matrices is NOT diagonalizable?
(A) $\begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$ (B) $\begin{pmatrix} 1 & 0 \\ 3 & 2 \end{pmatrix}$ (C) $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ (D) $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

23. Let V be the column space of the matrix $A = \begin{pmatrix} 1 & -1 \\ 1 & 2 \\ 1 & -1 \end{pmatrix}$. Then the orthogonal projection of $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ on
(A) $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ (B) $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ (C) $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ (D) $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

24. Let $\sum_{n=-\infty}^{\infty} a_n (z+1)^n$ be the Laurent series expansion of $f(z) = \sin\left(\frac{z}{z+1}\right)$. Then $a_{-2} =$
(A) 1 (B) 0 (C) $\cos(1)$ (D) $\frac{-1}{2}\sin(1)$

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25. Let u(x, y) be the real part of an entire function f(z) = u(x, y) + iv(x, y) for z = x + iy $\in$ C. If C is the positively oriented boundary of a rectangular region R in R^2, then $\oint_C \left[\frac{\partial u}{\partial y}dx - \frac{\partial u}{\partial x}dy\right] =$

(A) 1 (B) 0 (C) 2π (D) π

26. Let ϕ: [0, 1] $\to$ R be three times continuously differentiable. Suppose that the iterates defined by $x_{n+1} = \phi(x_n)$, n $\ge$ 0 converge to the fixed point ξ of ϕ. If the order of convergence is three then

(A) $\phi'(\xi) = 0, \phi''(\xi) = 0$ (B) $\phi'(\xi) \neq 0, \phi''(\xi) = 0$

(C) $\phi'(\xi) = 0, \phi''(\xi) \neq 0$ (D) $\phi'(\xi) \neq 0, \phi''(\xi) \neq 0$

27. Let f:[0, 2] $\to$ R be a twice continuously differentiable function. If $\int_0^2 f(x)dx \approx 2f(1)$, the error in the approximation is

(A) $\frac{f''(\xi)}{12}$ for some $\xi \in (0, 2)$ (B) $\frac{f''(\xi)}{2}$ for some $\xi \in (0, 2)$

(C) $\frac{f''(\xi)}{3}$ for some $\xi \in (0, 2)$ (D) $\frac{f''(\xi)}{6}$ for some $\xi \in (0, 2)$

28. For a fixed t $\in$ R , consider the linear programming problem:

Maximize z = 3x + 4y
Subject to x + y $\le$ 100
x + 3y $\le$ t
and x $\ge$ 0, y $\ge$ 0,

The maximum value of z is 400 for t =

(A) 50 (B) 100 (C) 200 (D) 300

29. The maximum value of $z = 2x_1 - x_2 + x_3 - 5x_4 + 22x_5$ subject to

$x_1 + x_4 + x_5 = 6$
$x_2 + x_4 - 4x_5 = 3$
$x_3 + 3x_4 + 2x_5 = 10$
$x_j \ge 0$, j = 1, 2........, 5 is

(A) 28 (B) 19 (C) 10 (D) 9

30. Using the Hungarian method, the optimal value of the assignment problem whose cost matrix is given by

5	23	14	8
10	25	1	23
35	16	15	12
16	23	11	7

is

(A) 29 (B) 52 (C) 26 (D) 44

31. Which of the following sequence $\{f_n\}_{n=1}^{\infty}$ of functions does NOT converge uniformly on [0, 1]?

(A) $f_n(x) = \frac{e^{-x}}{n}$ (B) $f_n(x) = (1-x)^n$ (C) $f_n(x) = \frac{x^2+nx}{n}$ (D) $f_n(x) = \frac{\sin(nx+n)}{n}$

32. Let E = {(x, y) $\in$ R^2: 0 < x < y}. Then $\iint_E ye^{-(x+y)}dxdy =$

(A) $\frac{1}{4}$ (B) $\frac{3}{2}$ (C) $\frac{4}{3}$ (D) $\frac{3}{4}$

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33. Let $f_n(x) = \frac{1}{n}\sum_{k=0}^{n}\sqrt{k(n-k)}\binom{n}{k}x^4(1-x)^{n-k}$ for $x \in [0, 1]$, n = 1, 2.... if $\lim_{n\to\infty} f_n(x) = f(x)$ for $x \in [0, 1]$, then the maximum value of f(x) on [0, 1] is

(A) 1 (B) $\frac{1}{2}$ (C) $\frac{1}{3}$ (D) $\frac{1}{4}$

34. Let f: $(c_{00}, \|\cdot\|_1) \to C$ be a non-zero continuous linear functional. The number of Hahn-Banach extensions of f to $(\ell^1, \|\cdot\|_1)$ is

(A) one (B) two (C) three (D) infinite

35. If I: $(\ell^1, \|\cdot\|_2) \to (\ell^1, \|\cdot\|_1)$ is the identity map, then

(A) both I and I^{-1} are continuous
(B) I is continuous but I^{-1} is NOT continuous
(C) I^{-1} is continuous but I is NOT continuous
(D) neither I nor I^{-1} is continuous

36. Consider the topology $\tau = \{G \subseteq R: R \setminus G$ is compact in $(R, \tau_u)\} \cup \{\phi, R\}$ on R, where τ_u is the usual topology on R and ϕ is the empty set. Then (R, τ) is

(A) a connected Hausdorff space
(B) connected but NOT Hausdorff
(C) Hausdorff but NOT connected
(D) neither connected nor Hausdorff

37. Let

$\tau_1 = \{G \subseteq R$: G is finite or $R\setminus G$ is finite$\}$

and

$\tau_2 = \{G \subseteq R$: G is countable or $R\setminus G$ is countable$\}$

Then

(A) neither τ_1 nor τ_2 is a topology on R
(B) τ_1 is a topology on R but τ_1 is NOT a topology on R
(C) τ_2 is a topology on R but τ_2 is NOT a topology on R
(D) both τ_1 and τ_2 are topology on R

38. Which one of the following ideals of the ring Z[i] of Gaussian integers is NOT maximal?

(A) $\langle 1 + i\rangle$ (B) $\langle 1 - i\rangle$ (C) $\langle 2 + i\rangle$ (D) $\langle 3 + i\rangle$

39. If Z(G) denotes the centre of a group G, then the order of the quotient group G/Z(G) cannot be

(A) 4 (B) 6 (C) 15 (D) 25

40. Let Aut(G) denote the group of automorphisms of a group G. Which one of the following is NOT a cycling group?

(A) Aut (Z_4) (B) Aut (Z_6) (C) Aut (Z_8) (D) Aut (Z_{10})

41. Let X be a non-negative integer valued random variable with $E(X^2) = 3$ and $E(X) = 1$. Then

$$\sum_{i=1}^{\infty} iP(X \geq i) =$$

(A) 1 (B) 2 (C) 3 (D) 4

42. Let X be a random variable with probability density function $f \in \{f_0, f_1\}$, where

$$f_0(x) = \begin{cases} 2x, \text{if } 0<x<1 \\ 0, \text{otherwise} \end{cases} \text{ and } f_1(x) = \begin{cases} 3x^2, \text{if } 0<x<1 \\ 0, \text{otherwise} \end{cases}$$

For testing the null hypothesis $H_0: f \equiv f_0$ against the alternative hypothesis $H_1: f \equiv$ at level of significance $\alpha = 0.19$, the power of the most powerful test is

(A) 0.729 (B) 0.271 (C) 0.615 (D) 0.385

43. Let X and Y be independent and identically distributed u(0, 1) random variables.

Then $P\left(Y,, < \left(x - \frac{1}{2}\right)^2\right) =$

(A) $\frac{1}{12}$ (B) $\frac{1}{4}$ (C) $\frac{1}{3}$ (D) $\frac{2}{3}$

44. Let X and Y be Banach spaces and let $T:X\to Y$ be a linear map. Consider the statements:

P: if $x_n \to x$ in X then $Tx_n \to Tx$ in Y.

Q: if $x_n \to x$ in X and $Tx_n \to y$ in Y then $Tx = y$.

Then

(A) P implies Q and Q implies P
(B) P implies Q but Q does not implies P
(C) Q implies P but P does not implies Q
(D) neither P implies Q nor Q implies P

45. If $y(x) = x$ is a solution of the differential equation $y'' = \left(\frac{2}{x^2} + \frac{1}{x}\right)(xy' - y) = 0$, $0 < x < \infty$, then its general solution is

(A) $(\alpha + \beta e^{-2x})x$ (B) $(\alpha + \beta e^{2x})x$ (C) $\alpha x + \beta e^{x}$ (D) $(\alpha e^{x} + \beta)x$

46. Let $P_n(x)$ be the Legendre polynomial of degree n such that $P_n(1) = 1$, n = 1, 2,...... If

$$\int_{-1}^{1}\left(\sum_{j=1}^{n}\sqrt{J(2J+1)}p_J(X)\right)^2 dx = 20, \text{then } n =$$

(A) 2 (B) 3 (C) 4 (D) 5

47. The integral surface satisfying the equation $y\frac{\partial z}{\partial x} + x\frac{\partial z}{\partial y} = x^2 + y^2$ and passing through the curve $x = 1 - t$, $y = 1 + t$, $z = 1 + r^2$ is

(A) $z = xy + \frac{1}{2}(x^2 - y^2)^2$
(B) $z = xy + \frac{1}{4}(x^2 - y^2)^2$
(C) $z = xy + \frac{1}{8}(x^2 - y^2)^2$
(D) $z = xy + \frac{1}{16}(x^2 - y^2)^2$

48. For the diffusion problem $u_{xx} = u$, $(0 < x < \pi, t > 0)$. $u(0, t) = 0$, $u(\pi, t) = 0$ and $u(x, 0) = 3 \sin 2x$, the solution is given by

(A) $3e^{-t} \sin 2x$ (B) $3e^{-4t} \sin 2x$ (C) $3e^{-9t} \sin 2x$ (D) $3e^{-2t} \sin 2x$

49. A simple pendulum, consisting of a bob of mass m connected with a string of length a, is oscillating in a vertical plane. If the string is making an angle θ with the vertical, then the expression for the Lagrangian is given as

(A) $ma^2\left(\dot{\theta}^2 - \frac{2g}{a}\sin^2\left(\frac{\theta}{2}\right)\right)$
(B) $2mga\sin^2\left(\frac{\theta}{2}\right)$
(C) $ma^2\left(\frac{\dot{\theta}^2}{2} - \frac{2g}{a}\sin^2\left(\frac{\theta}{2}\right)\right)$
(D) $\frac{ma}{2}\left(\dot{\theta}^2 - \frac{2g}{a}\cos\theta\right)$

50. The extremal of the functional $\int_0^1\left(y + x^2 + \frac{y'^2}{4}\right)dx$, $y(0) = 0$, $y(1) = 0$ is

(A) $4(x^2 - x)$ (B) $3(x^2 - x)$ (C) $2(x^2 - x)$ (D) $x^2 - x$

Common Data Questions

Common Data Questions 51 and 52:

Let T: R^3 be the linear transformation defined by

$$T(x_1, x_2, x_3) = (x_1 + 3x_2 + 2x_3, 3x_1 + 4x_2 + x_3, 2x_1 + x_2 + x_3).$$

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51. The dimension of the range space of T^2 is
(A) 0 (B) 1 (C) 2 (D) 3

52. The dimension of the null space of T^3 is
(A) 0 (B) 1 (C) 2 (D) 3

Common Data for Questions 53 and 54:
Let $y_1(x) = 1 + x$ and $y_2(x) = e^x$ be two solutions of $y''(x) + P(x)\, y'(x) + Q(x)\, y(x) = 0$.

53. P(x) =
(A) $1 + x$ (B) $-1 - x$ 1 (C) $\frac{1+x}{x}$ (D) $\frac{-1-x}{x}$

54. The set of initial conditions for which the above differential equation has NO solution is
(A) $y(0) = 2, y'(0) = 1$ (B) $y(1) = 0, y'(1) = 1$
(C) $y(1) = 1, y'(1) = 0$ (D) $y(2) = 1, y'(2) = 2$

Common Data for Questions 55 and 56:
Let X and Y be random variables having the joint probability density function

$$f(x, y) = \begin{cases} \frac{1}{\sqrt{2\pi y}} e^{\frac{-1}{2y}(x-y)^2}, \text{ if } -\infty < x < \infty,\ 0 < y < 1 \text{ otherwise} \end{cases}$$

55. The variance of the random variable X is
(A) $\frac{1}{12}$ (B) $\frac{1}{4}$ (C) $\frac{7}{12}$ (D) $\frac{5}{12}$

56. The covariance between the random variables X and Y is
(A) $\frac{1}{3}$ (B) $\frac{1}{4}$ (C) $\frac{1}{6}$ (D) $\frac{1}{12}$

Linked Answer Questions 57 and 58:
Consider the function $f(z) = \frac{e^{iz}}{z(z^2+1)}$.

57. The residue of f at the isolated singular point in the upper half plane $\{z = x + iy \in C : y > 0\}$ is
(A) $\frac{-1}{2e}$ (B) $\frac{-1}{e}$ (C) $\frac{e}{2}$ (D) 1

58. The Cauchy principal value of the integral $\int_{-\infty}^{\infty} \frac{\sin x dx}{x(x^2+1)}$ is
(A) $-2\pi(1 + 2e^{-1})$ (B) $\pi(1 + e^{-1})$ (C) $2\pi(1 + e)$ (D) $-\pi(1 + e^{-1})$

Statement for Linked Answer Question 59 and 60:
Let $f(x, y) = kxy - x^3y - xy^3$ for $(x, y) \in R^2$, Where k is a real constant. The directional derivative of f at the point (1, 2) in the direction of the unit vector $u = \left(\frac{-1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$ is $\frac{15}{\sqrt{2}}$.

59. The value of k is
(A) 2 (B) 4 (C) 1 (D) −2

60. The value of *f* at a local minimum in the rectangular region $R = \left\{(x, y) \in R^2 : |x| < \frac{3}{2}, |y| < \frac{3}{2}\right\}$ is
(A) −2 (B) −3 (C) $\frac{-7}{8}$ (D) 0

End of question paper

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